Quantitative Timed Analysis of Interactive Markov Chains

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Why stochastic models?

- Random delays.
- External effects (like system inputs).
- Concurrent behavior of components.
- Reliability of components.
- Availability of components.

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expected time and long-run average

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	discrete-time	continuous-time
deterministic	DTMC	CTMC
non-deterministic	MDP	CTMDP IMC

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- Complexity of systems requires compositional approach
- Enormous model sizes require compositional abstraction mechanisms
- Nondeterminism is at heart of compositionality

Use of Interactive Markov Chains

Interactive Markov Chains (IMCs) are used for compositional semantics of:

- Architecture Analysis & Design Language (AADL)
- Dynamic fault trees (DFT)
- Generalized stochastic petri nets (GSPN)
- Scenario-aware dataflow languages
- Globally asynchronous locally synchronous hardware (GALS)
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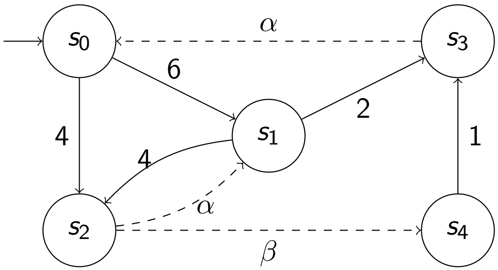
The foundation of IMCs are fundamental for these.

What are Interactive Markov Chains?

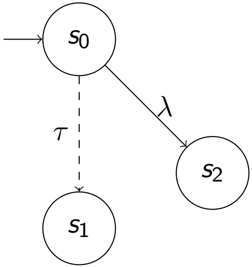
Interactive Markov chain

An interactive Markov chain is a tuple $\mathcal{I} = (S, Act, \dashrightarrow, \longrightarrow, s_0)$ where:

- S is a nonempty, finite set of states with *initial state* $s_0 \in S$,
- Act is a finite set of actions,
- $\dashrightarrow \subseteq S \times Act \times S$ is a set of *action* transitions and
- $\longrightarrow \subseteq S \times \mathbb{R}_{>0} \times S$ is a set of *Markovian* transitions.



Maximal progress



Note: The IMC is not subject to any further synchronisation.

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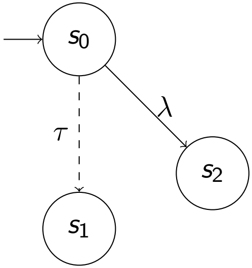
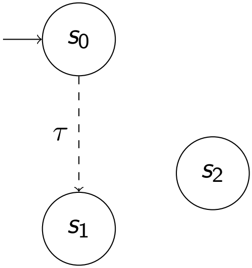


reduces to



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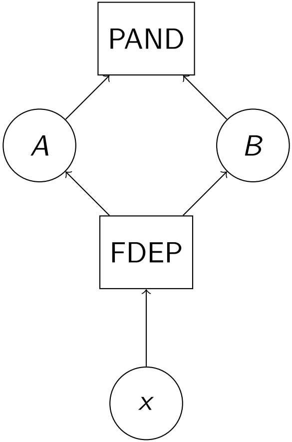
Dynamic Fault Tree



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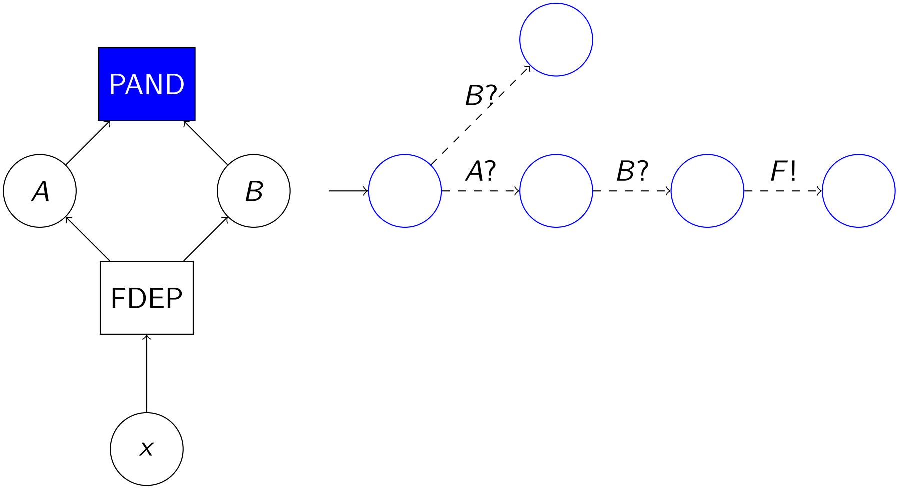

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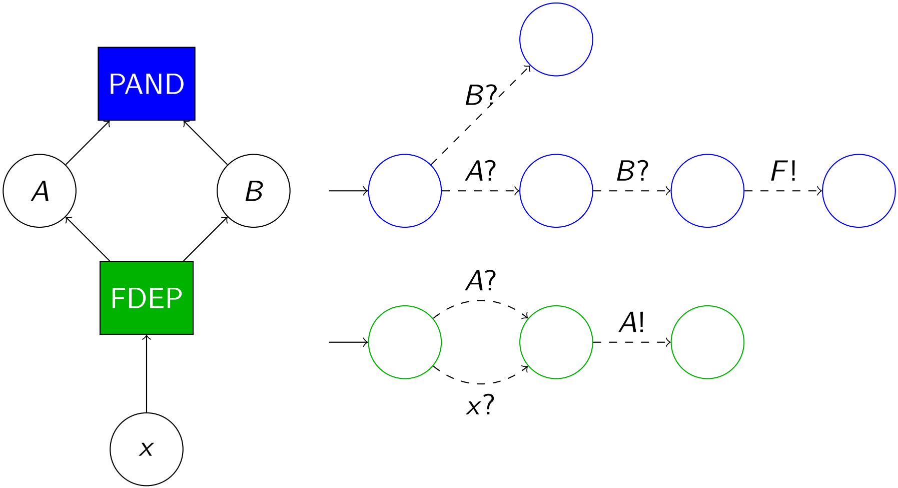

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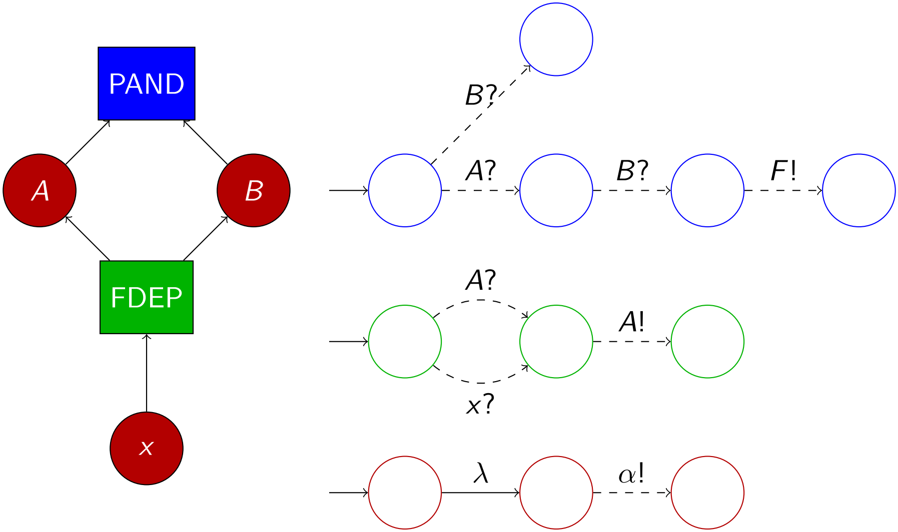

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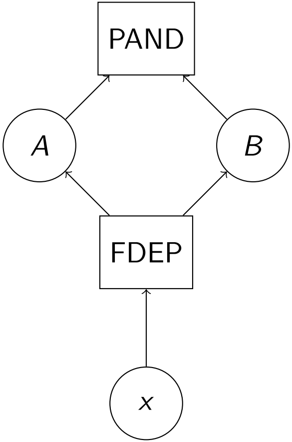

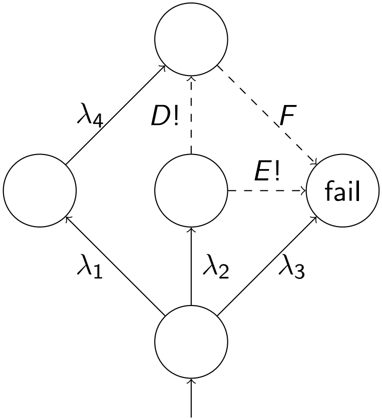

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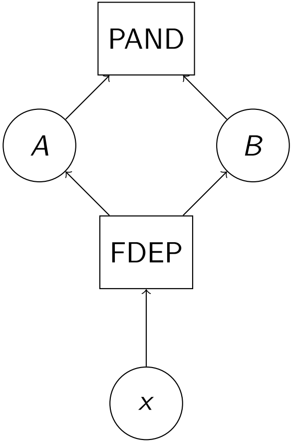

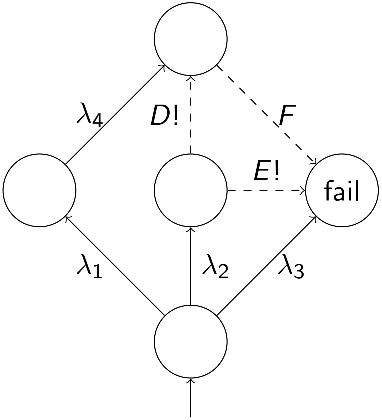

What is the expected time to fail?

Note: There is only the minimum and the maximum expected time due to nondeterminism.

Expected time

The elapsed time on a path before visiting some goal state in $G \subseteq S$.

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Minimal expected time

$$eT^{\min}(s, \Diamond G) = \inf_D \mathbb{E}_{s,D}(V_G) = \inf_D \int_{Paths} V_G(\pi) \Pr_{s,D}(d\pi).$$

minimum time t on an infinite path π, such that $G \cap \pi@t \neq \varnothing$

Note: Only the amount of time before entering a goal state in G is relevant.

Expected time

Function for minimal expected time

$$[L(v)]\,(s) = \begin{cases} \dfrac{1}{E(s)} + \displaystyle\sum_{s' \in S} \mathbf{P}(s,s') \cdot v(s') & \text{if } s \in MS \setminus G \\ \displaystyle\min_{s \xrightarrow{\alpha} s'} v(s') & \text{if } s \in IS \setminus G \\ 0 & \text{if } s \in G. \end{cases}$$

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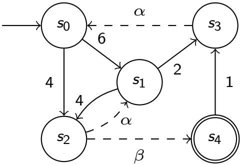
Corollary

$eT^{min}(s, \Diamond G)$ is the unique fixpoint of L.

Example



Determine
$eT^{min}(s_i, \Diamond\{s_4\})$

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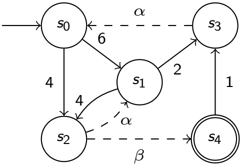

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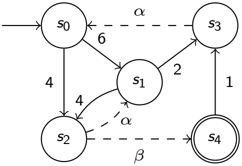

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- Maximize: $v(s_0) + v(s_1) + v(s_2) + v(s_3)$
  Subject to:

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\begin{aligned}
v(s_0) &\leq \frac{1}{10} + \frac{3}{5}v(s_1) + \frac{2}{5}v(s_2) \\
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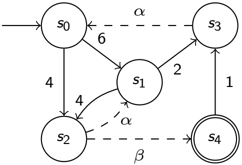

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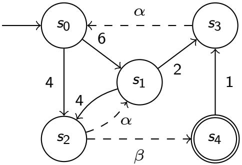

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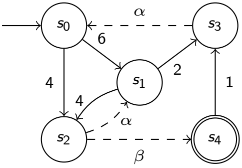

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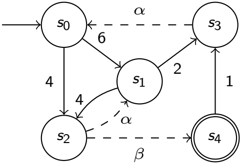

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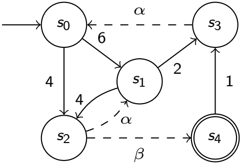

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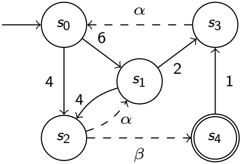

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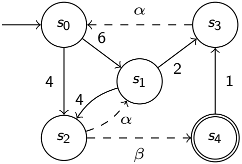

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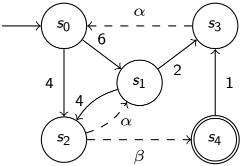

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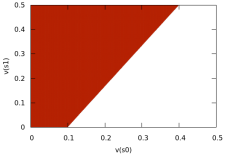

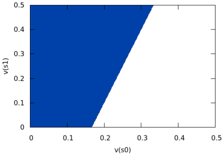

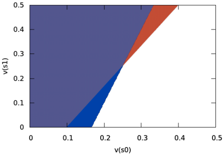

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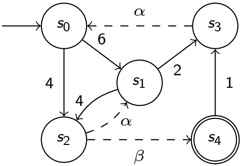

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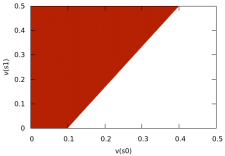

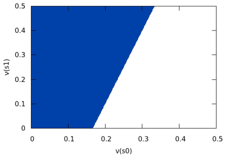

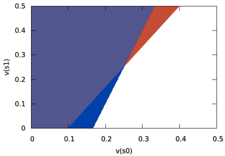

Long-run average

The amount of time we spent in $G \subseteq S$ in the long-run.

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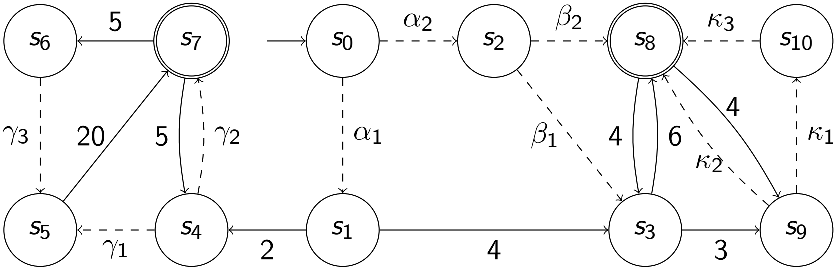
Long-run average

$$LRA^{D}(s, G) = \mathbb{E}_{s,D}(A_G) = \int_{Paths} A_G(\pi) \Pr_{s,D}(d\pi).$$

fraction of time spent in G on an infinite path π

Note: We search for the minimum $\inf_D$ or maximum $\sup_D$ long-run average.

Intuition


s_6
5
s_7
s_0
α_2
s_2
β_2
s_8
κ_3
s_{10}
γ_3
20
5
γ_2
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6
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s_3
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s_9

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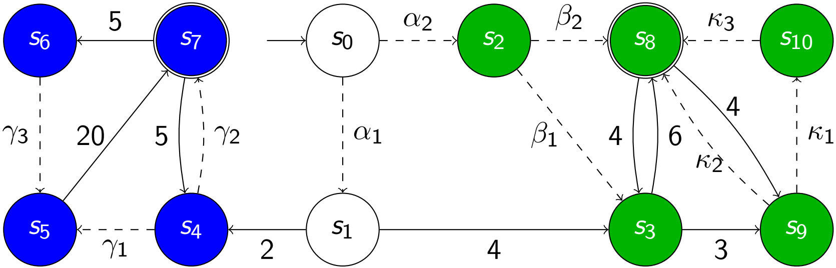

- Determine the maximal end components $\{\mathcal{I}_1, \ldots \mathcal{I}_k\}$ of IMC $\mathcal{I}$.
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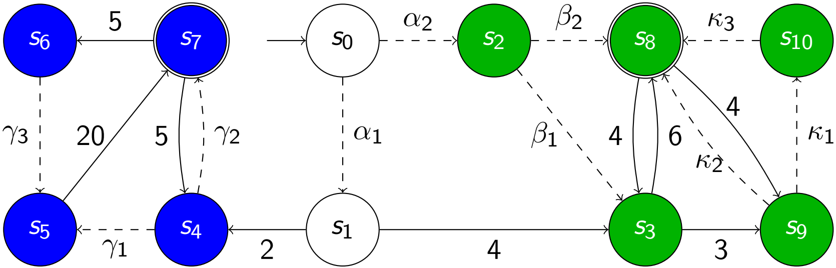

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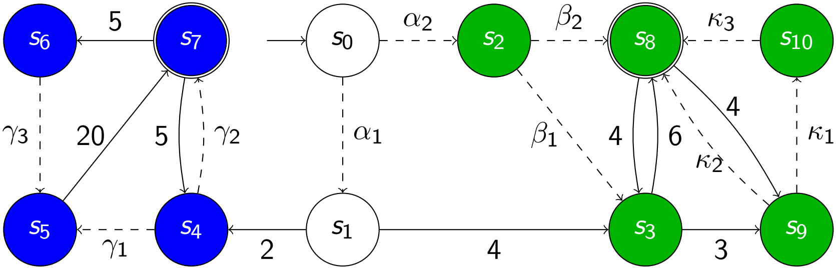


- Determine the maximal end components $\{\mathcal{I}_1, \dots \mathcal{I}_k\}$ of IMC $\mathcal{I}$.
- Determine $\text{LRA}(G)$ in maximal end components $\mathcal{I}_j$.
- Reduce the computation of $\text{LRA}(s_0, G)$ to an SSP problem.

Determine the long-run average

Long-run ratio

$$\mathcal{R}(\pi) = \lim_{n\to\infty} \frac{\sum_{i=0}^{n-1} c_1(s_i, \alpha_i)}{\sum_{j=0}^{n-1} c_2(s_j, \alpha_j)}.$$

$$c_1(s,\sigma) = \begin{cases} \frac{1}{E(s)} & \text{if } s \in MS \cap G \wedge \sigma = \perp \\ 0 & \text{otherwise,} \end{cases} \qquad c_2(s,\sigma) = \begin{cases} \frac{1}{E(s)} & \text{if } s \in MS \wedge \sigma = \perp \\ 0 & \text{otherwise.} \end{cases}$$

Determine the long-run average

Long-run ratio

$$\mathcal{R}(\pi) \;=\; \lim_{n\to\infty} \frac{\sum_{i=0}^{n-1} c_1(s_i,\alpha_i)}{\sum_{j=0}^{n-1} c_2(s_j,\alpha_j)}.$$

$$c_1(s,\sigma) \;=\begin{cases}\frac{1}{E(s)} & \text{if } s \in MS \cap G \wedge \sigma = \bot\\ 0 & \text{otherwise,}\end{cases} \qquad c_2(s,\sigma) \;=\begin{cases}\frac{1}{E(s)} & \text{if } s \in MS \wedge \sigma = \bot\\ 0 & \text{otherwise.}\end{cases}$$

The ratio between the average residence time for states in G against all states in S.

Determine the long-run average

Minimum long-run ratio for MDP

$$R^{\min}(s) \;=\; \inf_{D} \mathbb{E}_{s,D}(\mathcal{R}) \;=\; \inf_{D} \sum_{\pi \in Paths} \mathcal{R}(\pi) \cdot \Pr_{s,D}(\pi).$$

Determine the long-run average

Minimum long-run ratio for MDP

$$R^{\min}(s) = \inf_D \mathbb{E}_{s,D}(\mathcal{R}) = \inf_D \sum_{\pi \in Paths} \mathcal{R}(\pi) \cdot \Pr_{s,D}(\pi).$$

Theorem

For unichain IMC $\mathcal{I}$, $\textsc{Lra}^{min}(s, G)$ equals $R^{min}(s)$ in MDP $\mathcal{M}(\mathcal{I})$.

Transform into linear programming problem

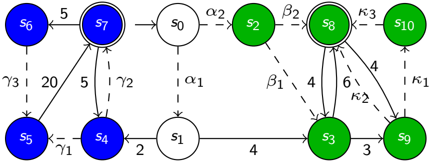
Use real variables k for the long-run and x_s for each $s \in S$:

Maximize k
Subject to:

$$x_s \leqslant c_1(s,\alpha) - k \cdot c_2(s,\alpha) + \sum_{s' \in S} \mathbf{P}(s,\alpha,s') \cdot x_{s'}.$$

for each $s \in S$ and $\alpha \in Act$.

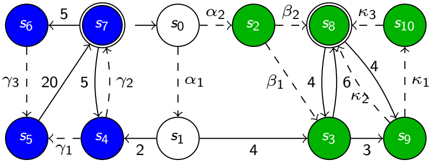
Example



$G = \{s_7, s_8\}$
Determine
$\mathrm{LRA}^{min}(s_0, G)$

-
-
-

Example



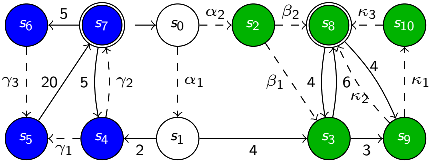
$G = \{s_7, s_8\}$
Determine
$\text{LRA}^{min}(s_0, G)$

- $\mathcal{I}_1$ with state space $S_1 = \{s_4, s_5, s_6, s_7\}$
-
-

Maximize: k
Subject to:

$$
\begin{aligned}
x_{s_4} &\leq x_{s_5} \\
x_{s_4} &\leq x_{s_7} \\
x_{s_5} &\leq -\frac{1}{20}k + x_{s_7} \\
x_{s_6} &\leq x_{s_5} \\
x_{s_7} &\leq \frac{1}{10} - \frac{1}{10}k + \frac{1}{2}x_{s_4} + \frac{1}{2}x_{s_4}
\end{aligned}
$$

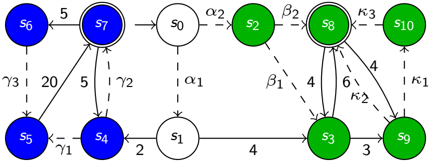
Example



$G = \{s_7, s_8\}$
Determine
$\text{LRA}^{min}(s_0, G)$

- $\mathcal{I}_1$ with state space $S_1 = \{s_4, s_5, s_6, s_7\}$
  $\text{LRA}_1^{min}(G) = \frac{2}{3}$
-
-

Example



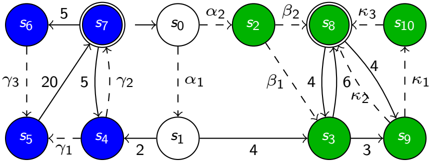
- $\mathcal{I}_1$ with state space $S_1 = \{s_4, s_5, s_6, s_7\}$
  $\text{LRA}_1^{min}(G) = \frac{2}{3}$
- $\mathcal{I}_2$ with state space $S_1 = \{s_2, s_3, s_8, s_9, s_{10}\}$
-

$G = \{s_7, s_8\}$
Determine
$\text{LRA}^{min}(s_0, G)$

Maximize: k
Subject to:

$$
\begin{aligned}
x_{s_2} &\leq x_{s_3} \\
x_{s_2} &\leq x_{s_8} \\
x_{s_3} &\leq -\tfrac{1}{9}k + \tfrac{2}{3}x_{s_8} + \tfrac{1}{3}x_{s_9} \\
x_{s_8} &\leq \tfrac{1}{8} - \tfrac{1}{8}k + \tfrac{1}{2}x_{s_3} + \tfrac{1}{2}x_{s_9} \\
x_{s_9} &\leq x_{s_8} \\
x_{s_9} &\leq x_{s_{10}} \\
x_{s_{10}} &\leq x_{s_3}
\end{aligned}
$$

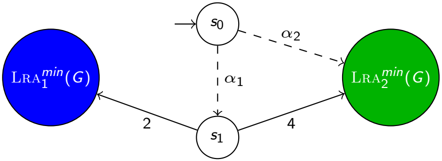
Example



$G = \{s_7, s_8\}$
Determine
$\text{LRA}^{min}(s_0, G)$

- $\mathcal{I}_1$ with state space $S_1 = \{s_4, s_5, s_6, s_7\}$
  $\text{LRA}_1^{min}(G) = \frac{2}{3}$
- $\mathcal{I}_2$ with state space $S_1 = \{s_2, s_3, s_8, s_9, s_{10}\}$
  $\text{LRA}_2^{min}(G) = \frac{9}{13}$
-

Example



$G = \{s_7, s_8\}$
Determine
$\text{LRA}^{min}(s_0, G)$

- $\mathcal{I}_1$ with state space $S_1 = \{s_4, s_5, s_6, s_7\}$
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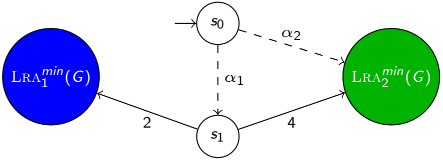
  $\text{LRA}_2^{min}(G) = \frac{9}{13}$
- Solving SSP problem:

Example



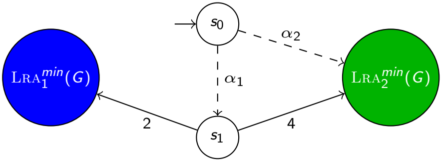
$G = \{s_7, s_8\}$
Determine
$\text{LRA}^{min}(s_0, G)$

- $\mathcal{I}_1$ with state space $S_1 = \{s_4, s_5, s_6, s_7\}$
  $\text{LRA}_1^{min}(G) = \frac{2}{3}$
- $\mathcal{I}_2$ with state space $S_1 = \{s_2, s_3, s_8, s_9, s_{10}\}$
  $\text{LRA}_2^{min}(G) = \frac{9}{13}$
- Solving SSP problem:

Maximize: $x_{s_0} + x_{s_1}$
Subject to:

$$x_{s_0} \leq \frac{9}{13}$$
$$x_{s_0} \leq x_{s_1}$$
$$x_{s_1} \leq \frac{1}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{9}{13}$$

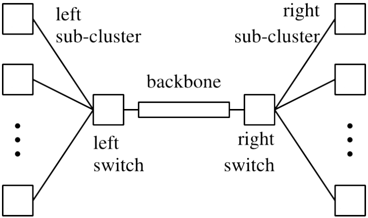
Example

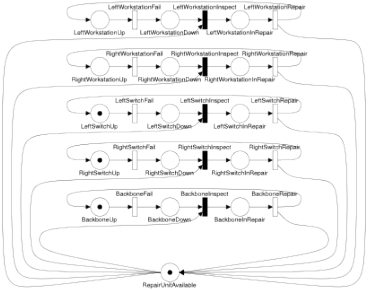


$G = \{s_7, s_8\}$
Determine
$\mathrm{LRA}^{min}(s_0, G)$

- $\mathcal{I}_1$ with state space $S_1 = \{s_4, s_5, s_6, s_7\}$
  $\mathrm{LRA}_1^{min}(G) = \frac{2}{3}$
- $\mathcal{I}_2$ with state space $S_1 = \{s_2, s_3, s_8, s_9, s_{10}\}$
  $\mathrm{LRA}_2^{min}(G) = \frac{9}{13}$
- Solving SSP problem:
  $\mathrm{LRA}^{min}(s_0, G) = \frac{80}{117}$

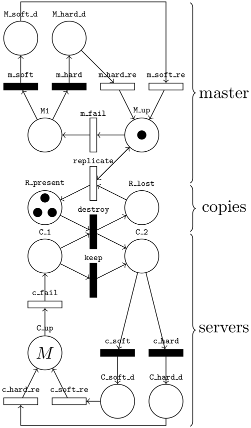
Workstation cluster (Haverkort, Hermanns & Katoen(2000))





GSPN of the workstation cluster as described in (Haverkort, Hermanns & Katoen(2000)).

Google file system (Cloth and Haverkort(2005))

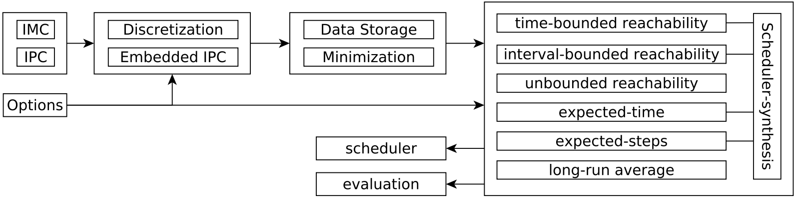


transition	rate / prob.
m_fail	0.0005
m_soft_re	#M_soft_d·12
m_hard_re	6.0
c_fail	0.05
c_soft_re	#C_soft_d·12
c_hard_re	1.0
replicate	20.0 if #R_present>0 2.0 if #R_present=0
m_soft	0.95
m_hard	0.05
c_soft	0.95
c_hard	0.05
destroy	$\frac{\#\texttt{R\_present}}{\#\texttt{C\_up}}$
keep	$1 - \frac{\#\texttt{R\_present}}{\#\texttt{C\_up}}$

Condition for replicate: $(\#\texttt{C\_up} > \#\texttt{R\_present}) \wedge (\#\texttt{C\_up} \cdot S \geqslant (\#\texttt{R\_present}+1) \cdot N)$

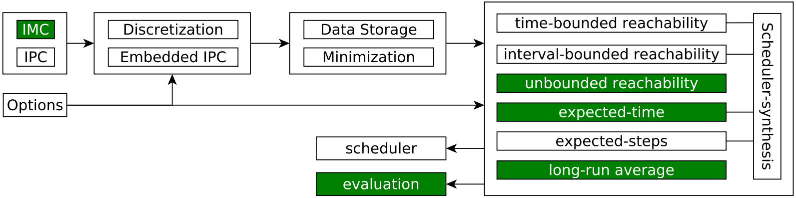
GSPN of the Google file system as described in (Cloth and Haverkort(2005)).

Interactive Markov Chain Analyzer (IMCA)



http://www-i2.informatik.rwth-aachen.de/imca/

Interactive Markov Chain Analyzer (IMCA)

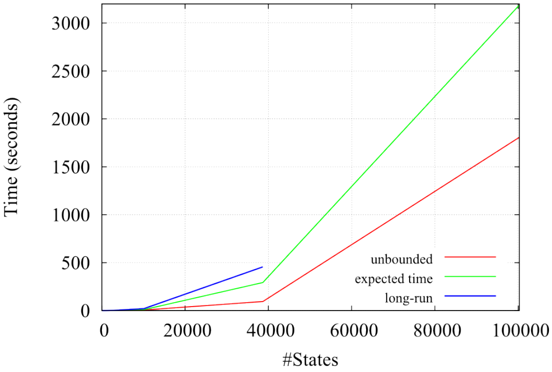


http://www-i2.informatik.rwth-aachen.de/imca/

Case studies (Workstation cluster)

N	# states	# transitions	$\|G\|$	$eT^{\max}(s, \Diamond G)$ time (s)	$\Pr^{\max}(s, \Diamond G)$ time (s)	$\text{LRA}^{\max}(s, G)$ time (s)
1	111	320	74	0.0009	0.0061	0.0046
4	819	2996	347	0.0547	0.0305	0.1137
8	2771	10708	1019	0.6803	0.3911	1.3341
16	10131	40340	3419	10.1439	5.3423	20.0278
32	38675	156436	12443	292.7389	94.0289	455.4387
52	100275	408116	31643	3187.1171	1807.7994	OOM

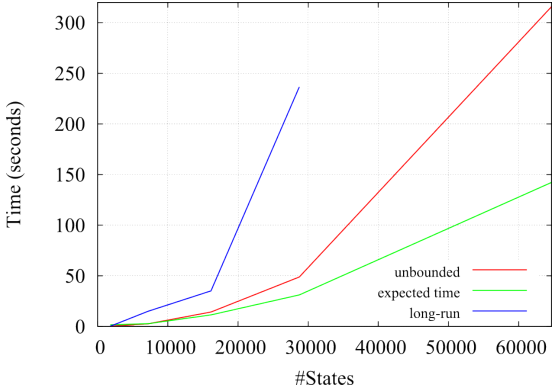
Computation times for the workstation cluster.



Case studies (Google file system)

M	# states	# transitions	$\lvert G \rvert$	$eT^{\min}(s, \Diamond G)$ time (s)	$\Pr^{\min}(s, \Diamond G)$ time (s)	$\textsc{Lra}^{\min}(s, G)$ time (s)
10	1796	6544	408	1.6568	0.1584	0.1411
20	7176	27586	1713	2.6724	2.5669	14.9804
30	16156	63356	3918	11.3836	14.2459	35.0654
40	28736	113928	7023	31.1416	48.8603	236.5308
60	64696	202106	15933	142.2179	315.8246	OOM

Computation times for Google file system ($S = 5000$ and $N = 100000$).



Conclusion

What we have seen

- Novel algorithms for expected time and long-run average
- Prototypical tool support in IMCA (http://www-i2.informatik.rwth-aachen.de/imca/)
- Both objectives can be reduced to SSP problems

What we investigate

- Generalization to Markov automata
  - combination of continous time and probabilistic transitions
- Experimentation with symbolic data structures such as BDDs
  - e.g. by exploiting PRISM for MDP analysis